II. Solution by BENJ. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, Ohio.

If not already so, any equation of the n^{th} degree may be reduced to the form $x^n+Ax^{n-1}+Bx^{n-2}+\ldots\ldots+L=0$. Now, by putting for x, $x+a$, we obtain a new equation whose roots differ from the corresponding roots of the given equation by a, (and whose degree, therefore, is still the n^{th}) viz.:

$$x^n+(na+A)x^{n-1}+\left(\frac{n(n-1)}{\underline{|2}}a^2+(n-1)Aa+B\right)x^{n-2}$$

$$+\ldots\ldots+(a^n+Aa^{n-1}+Ba^{n-2}+\ldots\ldots+L=0.$$

As a is an arbitrary constant, it may be selected so that $(na+A)=0$, or

$$\left(\frac{n(n-1)}{\underline{|2}}a^2+(n-1)Aa+B\right)=0,$$

or any coefficient, except the first, $=0$. Hence, any term, except the first, may thus be removed.

III. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Every algebraic equation may be written

$$X^k-\Sigma\alpha.X^{k-1}+\Sigma\alpha\beta.X^{k-2}\ldots\ldots=0.$$

The coefficient of the n^{th} term will be $\Sigma\alpha\beta\gamma\ldots\ldots$to $n-1$ factors. Now in place of X write $X+h$; then α, β, γ, etc., will be changed into $\alpha+h$, $\beta+h$, $\gamma+h$, etc. The coefficient of n^{th} term will then be $\pm\Sigma(\alpha+h)(\beta+h)(\gamma+h)\ldots\ldots$to $n-1$ terms. If we equate this to zero, we may consider it an equation of degree $n-1$ in h. This will give $n-1$ values of h. Therefore there are $n-1$ transformations which will make the n^{th} term vanish. Consider the first term, $n-1$; there are in that case no transformations.

Also solved by *PROF. E. W. MORRELL.*

63. Proposed by J. A. CALDERHEAD, A. B., Professor of Mathematics in Curry University, Pittsburg, Pennsylvania.

Given $x^2+x\sqrt{xy}=10$, and $y^2+y\sqrt{xy}=20$ to find x and y by quadratics.

I. Solution by E. L. BROWN, A. M., Professor of Mathematics, Capital University, Columbus, Ohio; HENRY HEATON, M. Sc., Atlantic, Iowa; and G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Factoring, we have $x^{\frac{3}{2}}(x^{\frac{1}{2}}+y^{\frac{1}{2}})=10$, $y^{\frac{3}{2}}(x^{\frac{1}{2}}+y^{\frac{1}{2}})=20$.

$$\therefore y^{\frac{3}{2}}/x^{\frac{3}{2}}=2,\ y^{\frac{3}{2}}=2x^{\frac{3}{2}}.\quad \therefore y=\sqrt[3]{4}x.$$

$\therefore\ y^{\frac{1}{2}}=\pm x^{\frac{1}{2}}\sqrt[3]{2}$, this in either equation gives

$$x^2(1\pm\sqrt[3]{2})=10,\quad \therefore\ x=\pm\sqrt{\frac{10}{1\pm\sqrt[3]{2}}},\ y=\pm\sqrt[3]{4}\sqrt{\frac{10}{1\pm\sqrt[3]{2}}}.$$

II. Solution by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania; COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee; and M. A. GRUBER, A. M., War Department, Washington, D. C.

Factoring the given equations, we obtain

$$x\sqrt{x}(\sqrt{x}+\sqrt{y})=10=a,\dots\dots\dots(1),\quad y\sqrt{y}(\sqrt{x}+\sqrt{y})=20=b,\ \dots\dots\dots(2).$$

$$(1)\div(2)\text{ gives }\frac{x\sqrt{x}}{y\sqrt{y}}=\frac{a}{b}.\ \text{ Squaring and reducing, we get}$$

$$y=\frac{x\sqrt[3]{b^2}}{\sqrt[3]{a^2}},\ \text{and}\ \sqrt{xy}=\frac{x\sqrt[3]{b}}{\sqrt[3]{a}}.$$

Substituting in first given equation, we have $x^2+\dfrac{x^2\sqrt[3]{b}}{\sqrt[3]{a}}=a$;

$$\text{whence } x=\pm\left(\frac{a\sqrt[3]{a}}{\sqrt[3]{a}+\sqrt[3]{b}}\right)^{\frac{1}{2}}=\pm\left(\frac{10}{1+\sqrt[3]{2}}\right)^{\frac{1}{2}},$$

$$\text{and } y=\pm\left(\frac{b\sqrt[3]{b}}{\sqrt[3]{a}+\sqrt[3]{b}}\right)^{\frac{1}{2}}=\pm\left(\frac{20\sqrt[3]{2}}{1+\sqrt[3]{2}}\right)^{\frac{1}{2}}.$$

III. Solution by BENJ. F. YANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, Ohio; and Prof. E. W. MORRELL, Montpelier Seminary, Montpelier, Vermont.

The given equations may be written,

$$x\sqrt{xy}=10-x^2\dots\dots\dots(1),\quad y\sqrt{xy}=20-y^2\dots\dots\dots(2).$$

$$(1)\times(2),\ x^2y^2=200-20x^2-10y^2+x^2y^2.\quad \therefore\ 2x^2+y^2=20\dots\dots\dots(3).$$

From (2) and (3), $2x^2=y\sqrt{xy}$. $\therefore\ y=x\sqrt[3]{4}\dots\dots\dots(4)$.

$$(4)\text{ in }(1),\ x=\pm\sqrt{\frac{10}{1+\sqrt[3]{2}}}.\quad \therefore\ y=\pm\sqrt{\frac{20\sqrt[3]{2}}{1+\sqrt[3]{2}}}.$$

IV. Solution by J. H. DRUMMOND, LL. D., Portland, Maine; A. H. HOLMES, Brunswick, Maine; and O. W. ANTHONY, M. Sc., New Windsor College, New Windsor, Maryland.

Let $y=v^2x$, then $x^2(1+v)=a=10$, and $v^3x^2(1+v)=b=20$.

$$\therefore\ v=\sqrt[3]{\frac{b}{a}},\ \text{and}\ x=\pm\frac{a^{\frac{2}{3}}}{\sqrt{a^{\frac{1}{3}}+b^{\frac{1}{3}}}},\ =\pm\left(\frac{10}{1+\sqrt[3]{2}}\right)^{\frac{1}{2}}.$$

$$y=\pm\frac{b^{\frac{2}{3}}}{\sqrt{a^{\frac{1}{3}}+b^{\frac{1}{3}}}},\ =\pm\left(\frac{20\sqrt[3]{2}}{1+\sqrt[3]{2}}\right)^{\frac{1}{2}}.$$

V. Solution by CHAS. A. HOBBS, A. M., Master of Mathematics in the Belmont School, Belmont, Massachusetts.

$$x^2+x^{\frac{3}{2}}y^{\frac{1}{2}}=10,\ y^2+x^{\frac{1}{2}}y^{\frac{3}{2}}=20.\quad \text{Let } y=vx.$$

$$\text{Then } x^2+v^{\frac{1}{2}}x^2=10,\ v^2x^2+v^{\frac{3}{2}}x^2=20.$$

$$\therefore\ x^2=\frac{10}{1+v^{\frac{1}{2}}},\ \text{and } x^2=\frac{20}{v^2+v^{\frac{3}{2}}}.\quad \therefore\ \frac{10}{1+v^{\frac{1}{2}}}=\frac{20}{v^2+v^{\frac{3}{2}}}.$$

Dividing by 10, and clearing of fractions, $v^{\frac{3}{2}}=2$, $v=2^{\frac{2}{3}}$.

$$\therefore\ x^2=\frac{10}{1+2^{\frac{1}{3}}},\ x=\sqrt{\frac{10}{1+\sqrt[3]{2}}}.\quad y=2^{\frac{2}{3}}\sqrt{\frac{10}{1+\sqrt[3]{2}}}=\sqrt{\frac{20\sqrt[3]{2}}{1+\sqrt[3]{2}}}.$$

VI. Solution by J. W. WATSON, Middle Creek, Ohio; and H. C. WILKES, Skull Run, West Virginia.

Put $x=m^2$, $y=n^2$. Then, the given equations become, after factoring, $m^3(m+n)=10$ ....(1), and $n^3(m+n)=20$........(2). Whence $n=m\sqrt[3]{2}$.

Then in (1) $m^3(m+m\sqrt[3]{2})=10$, or $m^4(1+\sqrt[3]{2})=10$.

$$\therefore\ m^4=\frac{10}{1+\sqrt[3]{2}},\ \text{and } m^2=\pm\sqrt{\frac{10}{1+\sqrt[3]{2}}},\ =x.$$

Also, n^2, $=y$, $=\pm\sqrt{\dfrac{20\sqrt[3]{2}}{1+\sqrt[3]{2}}}$.

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

56. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The locus of the centers of the isogonal transformations of all the diameters of the circumcircle of any triangle is the nine-points circle. *Brocard.*